Quantum nondemolition measurement of a light field component by a feedback compensated beam splitter

Holger F. Hofmann and Takayoshi Kobayashi
Department of Physics, Faculty of Science, University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo113-0033, Japan

Akira Furusawa
Department of Applied Physics, Faculty of Engineering, University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo113-8656, Japan
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Abstract

In conventional quantum nondemolition measurements, the interaction between signal and probe preserves the measured variable. Alternatively, it is possible to restore the original value of the variable by feedback. In this paper, we describe a quantum nondemolition measurement of a quadrature component of the light field using a feedback compensated beam splitter. The noise induced by the vacuum port of the beam splitter is compensated by a linear feedback resulting in an effective amplification of the observed variable. This amplification is then be reversed by optical parametric amplification to restore the original value of the field component.

The measurement backaction required by the uncertainty principle is a fundamental feature of quantum mechanics. In particular, this principle can be applied in quantum communications to prevent or detect eavesdropping [1]. However, most measurements in quantum optics are performed by irreversibly absorbing the measured light. It is therefore a special challenge to devise experimental realizations of back-action evasion measurements that allow the non-destructive observation of light field properties [2]. Usually, quantum nondemolition measurements of the light field are realized by a non-linear coupling between the signal field and a meter field [3–7]. In the following, a simple beam splitter will be used to couple the signal and the meter fields. While this method initially adds noise to the observed property of the signal field, it has been shown that this noise can be compensated by a measurement dependent feedback [8]. The result of this compensation is an amplification of the observed field component. By reversing this amplification in an optical parametric amplifier, the original value of the observable is restored. It is then possible to realize a quantum nondemolition measurement of a quadrature component using only one single mode parametric amplification step performed after the original beam splitter measurement. This approach thus represents a considerable simplification compared to the previous realizations of quadrature quantum nondemolition measurements which used two mode parametric amplification in order to couple the meter mode to the signal mode [5–7].

Figure 1 shows the experimental setup required for this quantum nondemolition measurement. The quantum nondemolition measurement is composed of a sequence of three distinct steps, the beam splitter measurement, the linear feedback, and the parametric amplification. In the first step, an unknown input state $\mid \psi_{\text{in}}\rangle$ is mixed with the vacuum at the beam splitter. The product state of the input state and the vacuum can be expressed in terms of the quadrature component x_S of the signal mode and the quadrature component x_M of the meter mode following the mixing of the modes at the beam splitter described by the unitary transformation $\hat{U}_{BS}$. For a reflectivity of $1-q^2$, the wavefunction of this quantum state reads

$$\langle x_S; x_M \mid \hat{U}_{BS} \mid \psi_{\text{in}}; \text{vac.}\rangle = \left(\frac{\pi}{2}\right)^{1/4} \exp\left[-(1-q^2)\left(x_S - \frac{q}{\sqrt{1-q^2}}x_M\right)^2\right] \langle \sqrt{1-q^2}\, x_M + q x_S \mid \psi_{\text{in}}\rangle. \quad (1)$$

It is then possible to obtain a measurement result x_m for the x-component of the input field by measuring the meter variable x_M, rescaling the result such that $x_m = x_M/\sqrt{1-q^2}$. The non-normalized output state in the signal field conditioned by this measurement reads

$$\mid \psi_{BS}(x_m)\rangle = \left(\frac{2(1-q^2)}{\pi}\right)^{1/4} \int dx_S\; \exp\left[-(1-q^2)\left(x_S - q x_m\right)^2\right] \langle (1-q^2)x_m + q x_S \mid \psi_{\text{in}}\rangle \mid x_S\rangle. \quad (2)$$

The measurement probability is given by $P(x_m) = \langle \psi_{BS}(x_m) \mid \psi_{BS}(x_m)\rangle$. The output amplitude of each component $\mid x_S\rangle$ now depends on the input amplitude at a x-value given by both x_S and the measurement result.

In the second step, the dependence of the output value of x on the noisy measurement result is compensated by feedback. This feedback can be described by a displacement operator $\hat{D}(\Delta x)$ with

$$\Delta x = \frac{1-q^2}{q} x_m. \quad (3)$$

The output state then reads

$$\hat{D}(\Delta x) \mid \psi_{BS}(x_m)\rangle = \left(\frac{2(1-q^2)}{\pi}\right)^{1/4} \int dx_S\; \exp\left[-\frac{1-q^2}{q^2}\left(q x_S - x_m\right)^2\right] \langle q x_S \mid \psi_{\text{in}}\rangle \mid x_S\rangle. \quad (4)$$

The output component $\mid x_S\rangle$ is now associated with the input amplitude of $\mid q x_S\rangle$. The vacuum noise component mixed into the signal by the beam splitter has been compensated completely, leaving only an effective amplification of the original field component x_S by a factor of $1/q$. This noiseless amplification has been realized experimentally by P.K. Lam and coworkers [8]. Equation (4) is a fully quantum mechanical formulation of this noiseless amplification setup.

In the final step of the quantum nondemolition measurement, the original value of x_S is restored by a parametric amplification described by the squeezing operator $\hat{S}(q)$ with

$$\hat{S}(q) \mid x_S\rangle = \sqrt{q} \mid q x_S\rangle. \quad (5)$$

This operation attenuates x_S back to its original value while simultaneously amplifying the conjugate quadrature component. The output state of this sequence of beam splitter measurement, feedback displacement, and parametric amplification is then given by

$$\hat{S}(q)\hat{D}(\Delta x) \mid \psi_{BS}(x_m)\rangle = \left(\frac{2(1-q^2)}{\pi\ q^2}\right)^{1/4} \int dx_S\ \exp\left[-\frac{1-q^2}{q^2}\left(x_S - x_m\right)^2\right] \langle x_S \mid \psi_{\text{in}}\rangle \mid x_S\rangle. \tag{6}$$

Note that this state is not normalized, because the measurement probabilities are given by $P(x_m) = \langle \psi_{BS}(x_m) \mid \psi_{BS}(x_m)\rangle$. Both the measurement statistics and the measurement back-action are thus described by equation (6). Since the output value of x_S is now equal to the input value, the measurement may be described by an operator commuting with the quadrature operator $\hat{x}_S$, such that

$$\hat{S}(q)\hat{D}(\Delta x) \mid \psi_{BS}(x_m)\rangle = \hat{P}_{\delta x}(x_m) \mid \psi_{\text{in}}\rangle. \tag{7}$$

The operator $\hat{P}_{\delta x}(x_m)$ is the generalized measurement operator for finite resolution quantum nondemolition measurements previously introduced in [9,10]. Expressed in terms of the quadrature operator $\hat{x}_S$, it reads

$$\hat{P}_{\delta x}(x_m) = \left(2\pi\delta x^2\right)^{-1/4} \exp\left[-\frac{(x_m - \hat{x}_S)^2}{4\delta x^2}\right]$$
$$\text{with} \quad 4\delta x^2 = \frac{q^2}{1-q^2}. \tag{8}$$

The probability distribution over measurement results $P(x_m)$ and the normalized output state $\mid \psi_{\text{out}}(x_m)\rangle$ are then given by

$$P(x_m) = \langle \psi_{\text{in}} \mid \hat{P}^2_{\delta x}(x_m) \mid \psi_{\text{in}}\rangle \tag{9}$$

$$\mid \psi_{\text{out}}(x_m)\rangle = \frac{1}{\sqrt{P(x_m)}} \hat{P}_{\delta x}(x_m) \mid \psi_{\text{in}}\rangle. \tag{10}$$

Note that the squared measurement resolution $4\delta x^2$ is given by the ratio of the transmission and the reflectivity of the beam splitter.

Since the measurement back-action represented by $\hat{P}_{\delta x}(x_m)$ is the minimum required by the uncertainty principle, the feedback compensated beam splitter represents an alternative realization of optimal quantum optical tapping [1,7]. The experimental effort should be greatly reduced by the use of a single mode parametric amplification instead of the two mode amplification applied in [6,7] according to the proposal of [5]. Moreover, the formalism developed above allows an assessment of a simple beam splitter measurement without feedback and of noiseless amplification [8] in terms of an operator product of the measurement operator $\hat{P}_{\delta x}(x_m)$ and the corresponding unitary transformations,

Beam splitter measurement:

$$\mid \psi_{BS}(x_m)\rangle \quad = \hat{D}(-\Delta x)\hat{S}(1/q)\hat{P}_{\delta x}(x_m) \mid \psi_{\text{in}}\rangle \tag{11}$$

Noiseless amplification:

$$\hat{D}(\Delta x) \mid \psi_{BS}(x_m)\rangle = \quad \hat{S}(1/q)\hat{P}_{\delta x}(x_m) \mid \psi_{\text{in}}\rangle. \tag{12}$$

Every step of the quantum nondemolition measurement can thus be described by its own operator. This allows an in depth analysis of the information and noise dynamics in quantum measurements. In particular, it should be noted that equation (11) represents the losses of the beam splitter without feedback in terms of a measurement dependent unitary transformation. The intensity loss caused by the beam splitter seems to originate from the dependence of the displacement operator on the classical information x_m. This indicates that the loss in intensity at the beam splitter is directly related to the classical information gain represented by the measurement result x_m. The implications for photon losses can be investigated by applying the operator formalism to single photon states or to low amplitude coherent states. The efficiency of photon loss compensation may be relevant for implementations of eavesdropping strategies for single photon quantum communication. An investigation of photon loss compensation should also provide practical insights into non-classical photon-field correlations [10], since the measurement only registers the reflected field amplitude, not the photon losses. By performing photon counting measurements on the transmitted signal field to monitor the changes in photon number, a simplified version of the experiment proposed in [10] can be realized.

In conclusion, we have described the realization of a quantum nondemolition measurement of a quadrature component by a feedback compensated beam splitter setup. This setup is more simple than previous realizations, since it requires only a single mode parametric amplification. The operator formalism describing the three sequential steps allows a theoretical investigation of the information dynamics in the quantum nondemolition measurement, in a beam splitter without feedback, and in noiseless amplification. Both the proposal for experiment and the formalism for its description should thus provide a helpful tool for the development of quantum communication technologies.

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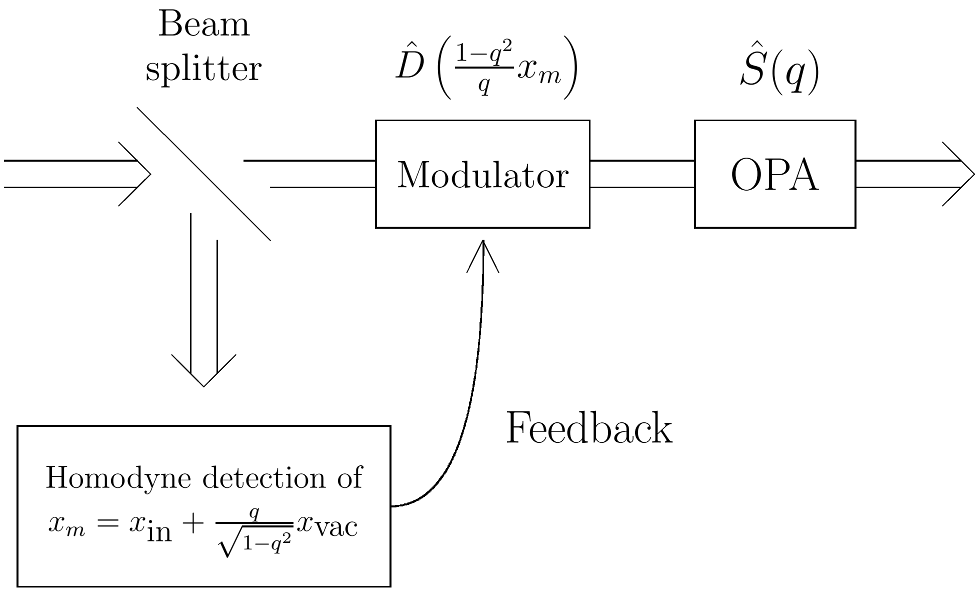

FIG. 1. Schematic setup of the quantum nondemolition measurement by feedback compensated beam splitter. The beam splitter reflectivity is $1 - q^2$. The measured component x_{in} mixes with the vacuum component x_{vac}, but the feedback compensates the contribution of x_{vac} in the output field.